Minimal subdynamics

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- More formally we say that a topological dynamical system G, X is a compact (metrizable) space X and a homomorphism from G to *homeo*(X)
- First example, circle rotation.
- Second example Shift action of Γ on 2^Γ The space $2^\Gamma = \{0,1\}^\Gamma$ consists of all labelings $x : \Gamma \to \{0,1\}$, equipped with the product topology (Cantor space).
  The <u>shift action</u> $\Gamma \curvearrowright 2^\Gamma$ is defined by

$$(\gamma \cdot x)(\delta) := x(\delta\gamma) \quad \text{for all } \gamma, \delta \in \Gamma,\ x \in 2^\Gamma.$$

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2. **Dense orbits:** For every $x \in X$, the orbit $\Gamma \cdot x$ is dense in X.
3. **Open sets are syndetic:** For every nonempty open set $U \subseteq X$, there exists a finite $F \subseteq \Gamma$ such that

$$F \cdot U = X.$$

(i.e. finitely many Γ-translates of U cover X.)

Observations about minimality

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1. **They exist** Every topological dynamical system has a minimal subsystem.
2. **Why** Zorn's lemma, alternatively, enumerate an open basis, iteratively remove if not syndetic
3. **Properties** Builds minimal systems, but gives you very little control over what they look like.

A Previously Open Question

Background

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e will actually ask a much more general question

S-minimality

Let $\Gamma \curvearrowright X$ be a continuous action on a compact Hausdorff space. How does it look like a dynamical systems for subgroups, for subsets? and let $S \subseteq \Gamma$.

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2. ***S*-syndetic open sets:** For every nonempty open $U \subseteq X$, there exists a finite $F \subseteq S$ such that

$$F^{-1} \cdot U = X.$$

   (i.e. finitely many S-translates of U cover X.)

Let $\Gamma \curvearrowright X$ be a continuous action on a compact Hausdorff space, and let $\mathcal{F}$ be a family of finite subsets of Γ.

Definition ($\mathcal{F}$-minimality)

The flow X is $\mathcal{F}$-minimal if for every nonempty open set $U \subseteq X$, there exists $F \in \mathcal{F}$ such that

$$F^{-1} \cdot U = X.$$

That is, finitely many translates of U drawn from some $F \in \mathcal{F}$ cover X.

Main Theorem

Theorem (Bernshteyn–F)

Let Γ be a countable group. If $(\mathcal{F}_n)_{n \in \mathbb{N}}$ is a sequence of unbounded families of finite subsets of Γ, then there exists a free Γ-flow that is $\mathcal{F}_n$-minimal for all $n \in \mathbb{N}$.

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Corollary

If Γ has only countably many locally finite subgroups there exists a free Γ-flow that is Δ-minimal for every infinite subgroup $\Delta \leq \Gamma$.

Corollary (Countably many complete sections)

Let $(B_n)_{n\in\mathbb{N}}$ *be Borel complete sections in* $\mathrm{Free}(2^\Gamma)$*, and let* $(F_n)_{n\in\mathbb{N}}$ *be finite subsets of* Γ *with* $\sup|F_n| = \infty$*. Then there exists* $x \in \mathrm{Free}(2^\Gamma)$ *such that*

$$x \in F_n \cdot B_n \quad \textit{for infinitely many } n.$$

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Descriptive Set-Theoretic Corollaries

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Corollary (Uniform trapping)

If $B \subseteq \mathrm{Free}(2^\Gamma)$ is a Borel complete section, then there exists $n \in \mathbb{N}$ such that for every finite $F \subseteq \Gamma$ with $|F| \geq n$, the set $F \cdot B$ traps a point (i.e. some orbit is contained in $F \cdot B$).

Proof Sketch: Uniform Trapping

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3. By Baire Category there exist some open subset U of X such that B intersect U is comeagre.
4. Minimality forces X to be equal to $F_n \cdot U$ for some n, thus a generic point will be trapped.

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Ideas in the Proof

Despite the basic problem being classical the tools are very new! Three key ingredients enter the construction:

1. **Existence via genericity** The desired topological dynamical systems form a dense G_δ set in a carefully chosen space of topological dynamical systems. We don't find them explicitly, we use Baire Category

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2. **Asymptotic separation index** Our spaces are constructed using Asymptotic separation index A tool from descriptive combinatorics inspired by large-scale geometry of graphs, this seems to be the first application of this tool to pure topological dynamics

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3. **Definable Lovász Local Lemmas** In order to prove the result we use use a continuous variant of the Lovász Local Lemmas to build approximations with the right properties.

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Why is this weird

What should actions on arbitrary polish spaces have to do with building TDS?, Hints that the dynamics on arbitrary polish spaces can still be very interesting!

Witnesses to ASI and Finite ASI

Space of witnesses.

Fix $s \in \mathbb{N}_{>0}$ and a finite $\Phi \subseteq \Gamma$. Define

$$\mathrm{Sep}(s, \Phi) \ := \ \left\{ x : \Gamma \to \{0, 1, \ldots, s\} \ \middle| \ \forall i, \ x^{-1}(i) \text{ is } \Phi\text{-finite} \right\}.$$

(meaning the Φ-connected components are finite.

Set

$$\mathrm{Sep}(s) \ := \ \prod_{n \in \mathbb{N}} \mathrm{Sep}(s, \Phi_n),$$

where (Φ_n) enumerates all finite subsets of Γ. $\mathrm{Sep}(s)$ carries the diagonal shift action

$$(\gamma \cdot x)_n(\delta) \ := \ x_n(\delta\gamma) \qquad (\gamma, \delta \in \Gamma).$$

Finite (continuous) ASI.

For a free continuous action $\Gamma \curvearrowright X$ on a zero-dimensional Polish space, the continuous asymptotic separation index $\mathrm{asi}(X)$ is the least s such that X can be mapped continously into $Sep(s)$

Amply Syndetic Spaces

Definition (Amply syndetic Γ-space)

A Polish Γ-space X is amply syndetic if:
For every finite tuple of nonempty open sets $U_1, \ldots, U_k \subseteq X$, there exists $n \in \mathbb{N}$ such that for every finite $F \subseteq \Gamma$ with $|F| \geq n$, there is a continuous Γ-equivariant map $\pi : X \to X$ with the property that

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For any $s \in \mathbb{N}_{>0}$, the space $\mathrm{Sep}(s)$ of asymptotic s-separators is an amply syndetic Γ-space. The free part is a free-amply syndetic space.

Genericity Theorem

Theorem

Let X be a free amply syndetic Γ-space, and let Y be a free Polish Γ-flow.
Define

$$S = \overline{\{\, \rho(X) : \rho : X \to Y \text{ continuous and } \Gamma\text{-equivariant}\,\}}.$$

For every unbounded family $\mathcal{F}$ of finite subsets of Γ, the set of subshifts $Z \in \overline{S}$ that are $\mathcal{F}$-minimal forms a dense G_δ subset of $\overline{S}$.

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Topology on $\overline{S}$

We view $\overline{S}$ as a subspace of the space of closed Γ-invariant subsets of Y, equipped with the Vietoris topology.

Sketch of the Argument

Goal

Show that $\mathcal{F}$-minimal subshifts are dense G_δ in $\overline{S}$.

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Idea.

1. G_δ is easy to see. So the work is density
2. Fix a nonempty open set $\mathcal{O} \subseteq \overline{S}$ in the Vietoris topology.
3. By definition of $\overline{S}$, $\mathcal{O}$ contains some subflow close to an image $\rho(X)$.
4. Use the amply syndetic property of X: given finitely many open sets O_n in Y, we can find a Γ-equivariant map $\pi : X \to X$ such that $\pi^{-1}(\rho^{-1}(O_n))$ are all $\mathcal{F}$-syndetic.
5. Thus each O_n shows up $F -$ *syndetically*. Thus $\rho\pi$ gives up the appropriate approximation map

□

Thank you

- Thank you to the organizers!

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- Thank you for listening!